

Mathematical Methods.

Problem set 5. Hand-in 20/10-2008

1. Show that

$$a) \int_0^{2\pi} \frac{\sin(3\theta)}{5 - 3\cos(\theta)} d\theta = 0, \quad b) \int_{-\infty}^{\infty} \frac{dx}{(x^2 + 4x + 5)^2} = \frac{\pi}{2}.$$

2. Find the radii of convergence for the three power series

$$a(z) = \sum_n z^n, \quad b(z) = \sum_n i^{n+1}(z - i - 1)^n, \quad c(z) = \sum_n (-1)^{n+1}(z - 2)^n.$$

Sketch the overlaps of the discs and show that the series are equal there.

3. Describe the cuts, branches and Riemann surfaces for the multiform functions

$$a) \sqrt{z}, \quad b) (z + 43)^{1/7}, \quad c) \sqrt{1 - z^2},$$

4. Find the general form of the Möbius transformation which.

- a) Maps the upper half plane to itself.
- b) Maps the upper half plane to the lower half plane.
- c) Maps the upper half plane to the right half plane.
- d) Preserves the unit circle.
- e) Preserves the coordinate axes.

5. Find the most general cubic polynomial

$$u(x, y) = ax^3 + bx^2y + cxy^2 + dy^3$$

that is harmonic. Find a differentiable function of which u is the real part.

6. (Hard! See how far you get...) Show that

$$\int_0^1 \frac{dx}{(1 + ax^2)\sqrt{1 - x^2}} = \frac{\pi}{2\sqrt{1 + a}}, \quad a > 0.$$