# Mathematical Methods.

**Problem set 8.**
**Hand-in 10/11-2008**

1. Show that the following functions obey Laplace's equation in 2 dimensions and draw sketch equipotential lines,

$$a)\;\; u(x,y) = xy, \quad b)\;\; u(x,y) = x^2 - y^2, \quad c)\;\; (x^2 - y^2)/(x^2 + y^2)^2$$

2. Show that the following functions are solutions to the heat equation

$$a)\;\; u(x,t) = e^{-2t}\cos(x), \quad b)\;\; u(x,t) = e^{-t}\sin(3x), \quad c)\;\; u(x,t) = e^{-4t}\cos(\omega x).$$

3. Show that the following functions are solutions to the wave equation in 1+1D,

$$a)\;\; u(x,t) = x^2 + 4t^2, \quad b)\;\; u(x,t) = x^3 + 3xt^2, \quad c)\;\; u(x,t) = \sin(\omega c t)\sin(\omega x)$$

4. Show that $u(x,y) = a\ln(x^2 + y^2) + b$ solves the Laplace equation and determine $a, b$ from the boundary conditions

$$u = 0, \quad x^2 + y^2 = 1 \quad \text{and} \quad u = 5, \quad x^2 + y^2 = 9.$$

5. Solve the Laplace equation in spherical coordinates with the boundary conditions on the unit sphere given by

$$a)\;\; f(\theta,\phi) = 1, \quad b)\;\; f(\theta,\phi) = \cos(\theta), \quad c)\;\; f(\theta,\phi) = \cos(2\theta), \quad d)\;\; f(\theta,\phi) = \cos(3\theta) + 3\cos(\theta).$$

6. Using separation of variables, find the general solution of

$$\frac{\partial^2 u(x,t)}{\partial t^2} + c^2 \frac{\partial^4 u(x,t)}{\partial x^4} = 0,$$