

# Mathematical Methods.

## Problem set 9. Hand-in 17/11-2008

1. By using the indicated change of variables, solve the following differential equations

$$\begin{aligned} a) \quad & u_{xy} - u_{yy} = 0, \quad v = x, \quad z = x + y, \\ b) \quad & xu_{xy} = yu_{yy} + u_y, \quad v = x, \quad z = xy, \\ c) \quad & u_{xx} + u_{xy} - 2u_{yy} = 0, \quad v = x + y, \quad z = 2x - y \end{aligned}$$

How does this look in terms of matrix notation and coordinate transformations?

2. Differential equation of the type

$$Au_{xx} + 2Bu_{xy} + Cu_{yy} = 0,$$

can be *elliptic* ( $AC - B^2 > 0$ ), *parabolic* ( $AC - B^2 = 0$ ), *hyperbolic* ( $AC - B^2 < 0$ ). What types are the wave, heat and Laplace's equations? What about the Tricomi equation

$$yu_{xx} + u_{yy} = 0.$$

3. Solve the two-dimensional wave equation for a rectangular membrane  $[0; a] \times [0; b]$ , with zero initial velocity and initial displacement

$$a) \quad f(x, y) = 1, \quad b) \quad f(x, y) = x + y, \quad c) \quad f(x, y) = xy(a^2 - x^2)(b^2 - y^2).$$

4. (Harder, extra! May need Mathematica and a bunch of Bessel functions) Solve the two dimensional wave equation for a circular membrane of radius  $R = 1$  with velocity of sound  $c = 1$ , with initial zero velocity and initial displacement

$$f(x, y) = k(1 - r^2).$$