Note added to sect. 2.6

Why does this work? Even though

$$\langle f^2 \rangle_N = \frac{1}{N} \sum_i f_i^2 \,,$$

does not have a well-defined limit as $N \to \infty$, the error estimate

$$\sigma_N = \sqrt{\frac{\langle f^2 \rangle_N - \langle f \rangle_N^2}{N-1}}$$

does, becaues of the extra factor of $(N-1)$ in the denominator! Thus, usually one can use the formula above as long as $f(x)$ is integrable. However, the convergence of the error is *slower than* $1/\sqrt{N}$, depending on the function!

What about integrals with non-integrable subdivergences? Example:

$$\int_{-1}^{1} dx \frac{1}{x} = 0$$

(in principal value sense). Usually Monte Carlo methods are not able to handle these (test the above!). The integral requires special treatment of the subdivergences (not discussed here).