Quantum Field Theory Spring 2008 Problem set 5, session 27.3, return by 1.4.

1. Using the formal integral identity

$$\int \mathcal{D}\phi \, \frac{\delta}{\delta\phi(x)} f(\phi) = 0$$

(where we assume boundary terms vanish), derive the so-called Schwinger-Dyson equation

$$0 = \left[\mathcal{L}'\left(\frac{\delta}{i\delta J(x)}\right) + J(x)\right] Z[J] \, .$$

Here $\mathcal{L}'(\phi) = \delta\mathcal{L}/\delta\phi$, or more precisely

$$\mathcal{L}'(\phi(x))\delta^4(x-y) = \frac{\delta\mathcal{L}(\phi(x))}{\delta\phi(y)} .$$

2. Consider the theory

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{3!}g\phi^3 .$$

where ϕ is a scalar field. What is the (free) Feynman propagator of this theory? Write down the expression for

$$Z[J] = e^{i\,S_I[\delta/(i\delta J)]} Z_0[J]$$

to order g, as was done in page 60 for $\lambda\phi^4$ theory. Calculate explicitely the expressions for all (connected) Green functions to order g^0 and g^1, following pages 61 and 62 (you will get only 1 diagram for $G^{(2)}$ and $G^{(3)}$, respectively).

From this you can already "guess" the Feynman rules for this theory, in analogy to (2.45). Using these write down the *connected* Feynman diagrams contributing to 2,3 and 4-point functions up to order g^2. What are the symmetry factors for these diagrams? (Symmetry factors work analogously to $\lambda\phi^4$-theory.)

3. Loop integral calculations can often be done with the help of Feynman parametization. It starts from the identity

$$\frac{1}{ab} = \int_0^1 \frac{d\alpha}{[a\alpha + b(1-\alpha)]^2}, \qquad a, b > 0.$$

Show that the above identity is true.