Quantum Field Theory Spring 2008 Problem set 6, session 3.4, return by 9.4.

1. Consider the 1-loop correction to the connected 4-point function discussed at the lectures. Show that, if the momenta of the external legs are not zero, the loop integral is of form (euclidean!)

$$I(q^2, m^2) = \int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2 + m^2} \frac{1}{(p+q)^2 + m^2}$$

   What is q in terms of the external momenta? Show that the integral is a function of $q^2 = \sum_\mu q_\mu^2$ only, and not the components of the vector q.

2. In the previous problem session we derived the Feynman parametrization

$$\frac{1}{ab} = \int_0^1 \frac{d\alpha}{[a\alpha + b(1-\alpha)]^2}, \qquad a, b > 0.$$

   Apply this to $I(q^2, m^2)$ in 1. and calculate I as far as you can. *Hint:* after applying the Feynman trick, consider a change of the integration variable $p' = p + (1-\alpha)\, q$. You should be able to do first the p' integral (following (3.43) in the notes), then the α integral.

3. Determine the integral

$$\int \frac{d^d p}{(2\pi)^d} \frac{1}{(p^2 + m^2)^n}$$

   n integer, in dimensional regularization. Using $d = 4 - 2\epsilon$, how does this behave at different n as $\epsilon \to 0$?