Quantum Field Theory Spring 2008 Problem set 7, session 10.4, return by 16.4.

1. Solve the differential equation

$$\mu \frac{d}{d\mu}\lambda_R = \frac{3}{(4\pi)^2}\lambda_R^2$$

by separation of variables.

2. Let us show now that $Z_\phi = 1 + \mathcal{O}(\lambda_R^2)$. In sec. 3.4. when we discussed the renormalization of the 2-point function we ignored Z_ϕ, which is justified only a posteriori. Using (3.40) we can write

$$\tilde{G}^{(2)}_{B,\bar{c}}(p) = Z_\phi^{-1}\tilde{G}^{(2)}_{R,\bar{c}}(p)$$

from which follows

$$\frac{1}{p^2 + m_B^2 + \Pi_B} = Z_\phi^{-1}\frac{1}{p^2 + m_R^2 + \Pi_R}$$

where Π_B is given in (3.25) and Π_R is the Π_B with the divergence subtracted. Consider the behaviour of the above equation at small momenta p. What would happen if Π had p-dependence (this occurs at 2-loop order)?

3. Using the normalization in the lecture notes, show that

$$u^\dagger(p,s)u(p,s') = E_{\vec{p}}\delta_{s,s'}$$