Quantum Field Theory Spring 2008 Problem set 8, session 17.4, return by 23.4.

1. Using $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ and the cyclic property of trace, show that
   a) $\mathrm{Tr}\,[\gamma^5] = 0$
   b) $\mathrm{Tr}\,[\text{odd number of } \gamma\text{'s}] = 0$
   c) $\mathrm{Tr}\,[\gamma^5\gamma^\mu] = 0$
   d) $\mathrm{Tr}\,[\gamma^5\gamma^\mu\gamma^\nu] = 0$

2. Let $c = (c_1 \;\; c_2)^T$, $c^\dagger = (c_1^* \;\; c_2^*)$ be 2-component Grassmann vectors, and M a 2×2 complex matrix. Calculate the Grassmann integral

$$\int \mathrm{d}c_1^*\, \mathrm{d}c_1\, \mathrm{d}c_2^*\, \mathrm{d}c_2 \;\; \exp[-c^\dagger M c]$$

3. With c's and M as above, we define

$$\langle \ldots \rangle = \frac{\int \mathrm{d}c_1^*\, \mathrm{d}c_1\, \mathrm{d}c_2^*\, \mathrm{d}c_2 \;\; (\ldots) \exp[-c^\dagger M c]}{\int \mathrm{d}c_1^*\, \mathrm{d}c_1\, \mathrm{d}c_2^*\, \mathrm{d}c_2 \;\; \exp[-c^\dagger M c]}$$

   Calculate explicitly
   a) $\langle c_1 c_l^* \rangle$
   b) $\langle c_1 c_1 \rangle$
   c) $\langle c_1 c_2 c_1^* c_2^* \rangle$