Collection of formulas for "Quantum optics in electric circuits", that is available to you in the examination.

$$m\ddot{q} + m\gamma\dot{q} + kq = F. \tag{1}$$

$$\alpha = \frac{1}{\sqrt{2\hbar}}(\sqrt{m\omega_0}q + i\frac{p}{\sqrt{m\omega_0}}) \tag{2}$$

$$\begin{aligned} F(t) &= \sqrt{2\hbar m\omega_0}f(t), \\ f(t) &= 2f_0\cos(\omega_d t). \end{aligned} \tag{3}$$

$$\dot{\alpha} = -i\omega_0\alpha - \tfrac{1}{2}\gamma\alpha + if_0e^{-i\omega_d t}. \tag{4}$$

$$[a, a^\dagger] = 1, \quad a|n\rangle = \sqrt{n}|n-1\rangle, \quad a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle. \tag{5}$$

$$H = H_0 + H_\mathrm{d} + H_\mathrm{bath} + H_\mathrm{int} \tag{6}$$

$$H_0 = \hbar\omega_0(a^\dagger a + \frac{1}{2}) \tag{7}$$

$$H_\mathrm{d} = -\hbar(a + a^\dagger)f(t) \tag{8}$$

$$H_\mathrm{bath} = \sum_i \hbar\omega_i b_i^\dagger b_i \tag{9}$$

$$H_\mathrm{int} = -\hbar(a + a^\dagger)(\Gamma + \Gamma^\dagger), \qquad \Gamma = \sum_i g_i b_i. \tag{10}$$

$$i\hbar\frac{dA}{dt} = [A, H] + i\hbar\frac{\partial A}{\partial t}. \tag{11}$$

$$i\hbar\frac{d\rho}{dt} = [H, \rho], \qquad \langle A\rangle = \mathrm{Tr}(A\rho). \tag{12}$$

$$\rho = \frac{1}{Z}e^{-\beta H}, \qquad \beta = 1/k_BT. \tag{13}$$

$$\langle n\rangle = \frac{1}{e^{\beta\hbar\omega_0} - 1} \equiv N \tag{14}$$

$$\begin{aligned} \frac{d\rho}{dt} &= -\frac{i}{\hbar}[H_0 + H_\mathrm{d}, \rho] + \frac{\gamma}{2}(N+1)(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a) \\ &\quad + \frac{\gamma}{2}N(2a^\dagger\rho a - aa^\dagger\rho - \rho aa^\dagger) \end{aligned} \tag{15}$$

$D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$,
$e^{A+B} = e^A e^B e^{-[A,B]/2}$, if $[A, [A, B]] = [B, [A, B]] = 0$,
$|\alpha\rangle = D(\alpha)|0\rangle$.

$$E_b = \frac{Q_b^2}{2C} = \frac{C\dot{\Phi}_b^2}{2}, \qquad E_b = \frac{\Phi_b^2}{2L} = \frac{L\dot{Q}_b^2}{2}. \tag{16}$$

$$L = T - V, \qquad H = \sum_i \dot{q}_i p_i - L, \qquad [q_i, p_i] = i\hbar\delta_{ij}. \tag{17}$$

$$H(\Phi, Q) = \frac{(Q + Q_0)^2}{2C_\Sigma} - E_J\cos\frac{2\pi\Phi}{\Phi_0}, \qquad \Phi_0 = \frac{h}{2e}. \tag{18}$$

$$\begin{aligned} H &= \sum_n \Bigg[4E_c\left(n + \frac{Q_0}{2e}\right)^2 |n\rangle\langle n| \\ &\quad - \frac{E_J}{2}\left(|n+1\rangle\langle n| + |n-1\rangle\langle n|\right)\Bigg]. \end{aligned} \tag{19}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{20}$$

$$\rho = \frac{1}{2}(I + \boldsymbol{P}\cdot\boldsymbol{\sigma}) = \frac{1}{2}\begin{pmatrix} 1 + P_z & P_x - iP_y \\ P_x + iP_y & 1 - P_z \end{pmatrix}. \tag{21}$$

$$\begin{aligned} \frac{dP_x}{dt} &= (\boldsymbol{\Omega}\times\boldsymbol{P})_x - \frac{1}{T_2}P_x \\ \frac{dP_y}{dt} &= (\boldsymbol{\Omega}\times\boldsymbol{P})_y - \frac{1}{T_2}P_y \\ \frac{dP_z}{dt} &= (\boldsymbol{\Omega}\times\boldsymbol{P})_z - \frac{1}{T_1}(P_z - P_{z0}). \end{aligned} \tag{22}$$

$$\begin{aligned} \dot{u} &= -\Delta v - \frac{u}{T_2}, \\ \dot{v} &= \Delta u - \frac{v}{T_2} + \kappa\mathcal{E}w, \\ \dot{w} &= -\frac{w - w_\mathrm{eq}}{T_1} - \kappa\mathcal{E}v. \end{aligned} \tag{23}$$

$$\begin{aligned} u &= -w_\mathrm{eq}\frac{(\Delta T_2)(\kappa\mathcal{E}T_2)}{1 + (\Delta T_2)^2 + T_1T_2(\kappa\mathcal{E})^2}, \\ v &= w_\mathrm{eq}\frac{\kappa\mathcal{E}T_2}{1 + (\Delta T_2)^2 + T_1T_2(\kappa\mathcal{E})^2}, \\ w &= w_\mathrm{eq}\frac{1 + (\Delta T_2)^2}{1 + (\Delta T_2)^2 + T_1T_2(\kappa\mathcal{E})^2}. \end{aligned} \tag{24}$$

$$R_f(\tau) = \langle\delta f(t)\delta f(t - \tau)\rangle \tag{25}$$

$$S_f(\omega) = \int_{-T_w/2}^{T_w/2} d\tau e^{i\omega\tau}R_f(\tau) \tag{26}$$

$$S_F(\omega) = \frac{2\hbar\omega m\gamma(\omega)}{1 - e^{-\hbar\omega/k_BT}}. \tag{27}$$

$$H_{JC} = \frac{\hbar\Omega_0}{2}\sigma_z + \hbar\omega_0(a^\dagger a + \tfrac{1}{2}) - \hbar g(\sigma^+ a + \sigma^- a^\dagger), \tag{28}$$

$$\tilde{H} = \hbar\left(\omega_0 + \frac{g^2}{\Delta}\sigma_z\right)a^\dagger a + \frac{\hbar}{2}\left(\Omega_0 + \frac{g^2}{\Delta}\right)\sigma^z \tag{29}$$

$$\hat{f}(\omega) = \int_{-T_w/2}^{T_w/2} dt\, e^{i\omega t}f(t), \qquad f(t) = \frac{1}{T_w}\sum_{n=-\infty}^{\infty} e^{-i\omega_n t}\hat{f}(\omega_n). \tag{30}$$

$$G = \frac{2e^2}{h}M\mathcal{T}, \tag{31}$$

$$S_f(\omega) = \frac{1}{T_w}\langle\delta f(\omega)\delta f(-\omega)\rangle, \tag{32}$$

$$\begin{aligned} S_I(\omega) &= \frac{ge^2}{h}\int dE\int dE'\delta(\hbar\omega + E - E')|t|^2 \\ &\times\{|t|^2 f_\alpha(E)[1 - f_\alpha(E')] + |t|^2 f_\beta(E)[1 - f_\beta(E')] \\ &+ |r|^2 f_\alpha(E)[1 - f_\beta(E')] + |r|^2 f_\beta(E)[1 - f_\alpha(E')]\}. \end{aligned} \tag{33}$$

$\cosh x = (e^x + e^{-x})/2$, $\sinh x = (e^x - e^{-x})/2$, $\tanh x = 1/\coth x = \sinh x/\cosh x$.